The Dividend Discount Model
Adjusting Dividend Yield For Retained Free Cash Flow

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In this white paper we will solve for the dividend yield that allows for a portion of free cash flow to be retained and reinvested in an interest-earning cash account. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are tasked with solving for the dividend yield applicable to ABC Company given the following go-forward model assumptions...

Table 1: Go-Forward Model Assumptions

Symbol	Description	Value
C_0	Annualized free cash flow at time zero ($)	500,000
g	Cash flow growth rate (%)	5.00
r	Risk-free interest rate (%)	3.00
k	Risk-adjusted discount rate (%)	12.00

Our task is to answer the following question...

Question 1: What is total company value at the end of year 5 given that 60% of free cash flow is retained by the company and reinvested in a cash account earning the risk-free rate?

Question 2: What is the dividend yield that is consistent with the answer to the question above?

Building Our Model

Using the data in Table 1 above the equation for company value at time m via the dividend discount model is...

$$V_m = \frac{C_0\,(1+g)^{m+1}}{k-g} \tag{1}$$

Note that we can rewrite Equation (1) above as... [1]

$$V_m = \frac{C_0\,(1+g)}{k-g}\left(1+g\right)^m = V_0\left(1+g\right)^m \tag{2}$$

We defined the variable B_m to be the dividend cash account balance at time m and the variable Γ to be the portion of free cash flow retained by the company and deposited into the dividend cash account and reinvested. Using the data in Table 1 above the equation for the dividend cash account is... [1]

$$B_m = \Gamma\,V_0\left(k-g\right)\left(1+r\right)^m\left(1+g\right)^{-1}\left(\frac{\Pi-\Pi^{m+1}}{1-\Pi}\right) \text{ ...where... } \Pi = \frac{1+g}{1+r} \text{ ...and... } 0 \le \Gamma \le 1 \tag{3}$$

Note that the dividend discount model implies the following...

$$\text{capital gains} = g \text{ ...and... dividend income} = k-g \text{ ...such that... total return} = g+(k-g) = k \tag{4}$$

We will define the variable P_m to be total company value at time m, which is defined as company value via the dividend discount model plus the dividend cash account. Using Equations (2) and (3) above the equation for total company value is...

$$P_m = V_m + B_m \tag{5}$$

Note that we can rewrite Equation (6) above as...

$$P_m = V_0\bigg(1 + k - d\bigg)^m \quad \text{...where... } d = \text{DDM adjusted dividend yield} \tag{6}$$

If we equate Equations (5) and (6) above then the equation for that equality is...

$$V_m + B_m = V_0\bigg(1 + k - d\bigg)^m \tag{7}$$

Using Equatinos (2) and (3) above we can rewrite Equation (15) above as...

$$V_0\left(1 + g\right)^m + \Gamma\, V_0\bigg(k - g\bigg)\bigg(1 + r\bigg)^m\bigg(1 + g\bigg)^{-1}\left(\frac{\Pi - \Pi^{m+1}}{1 - \Pi}\right) = V_0\bigg(1 + k - d\bigg)^m \tag{8}$$

Using Equation (8) above and solving for the variable d, which is the DDM adjusted dividend yield, the equation for dividend yield is...

$$d = 1 + k - \left[\left(1 + g\right)^m + \Gamma\bigg(k - g\bigg)\bigg(1 + r\bigg)^m\bigg(1 + g\bigg)^{-1}\left(\frac{\Pi - \Pi^{m+1}}{1 - \Pi}\right)\right]^{\frac{1}{m}} \tag{9}$$

References

[1] Gary Schurman, *Retaining and Reinvesting Free Cash Flow,* March, 2021

Answers To Our Hypothetical Problem

Question 1: What is total company value at the end of year 5 given that 60% of free cash flow is retained by the company and reinvested in a cash account earning the risk-free rate?

Using Equation (1) above and the data in Table 1 above the equation for DDM company value at time zero is...

$$V_0 = \frac{500{,}000 \times (1 + 0.05)}{0.12 - 0.05} = 7{,}500{,}000 \tag{10}$$

Using Equation (2) above and the data in Table 1 above the equation for DDM company value at end of year 5 is...

$$V_5 = 7{,}500{,}000 \times (1 + 0.05)^5 = 9{,}572{,}100 \tag{11}$$

Using Equation (3) above and the data in Table 1 above the equation for the cash account value at end of year 5 is...

$$\begin{aligned} B_5 &= 0.60 \times 7{,}500{,}000 \times \bigg(0.12 - 0.05\bigg)\bigg(1 + 0.03\bigg)^5\bigg(1 + 0.05\bigg)^{-1}\left(\frac{1.0194 - 1.0194^6}{1 - 1.0194}\right) = 1{,}842{,}900 \\ &\text{where... } \Pi = \frac{1 + 0.05}{1 + 0.03} = 1.0194 \end{aligned} \tag{12}$$

Using Equations (5), (11) and (12) above the answer to the question is...

$$P_5 = 9{,}572{,}100 + 1{,}842{,}900 = 11{,}415{,}000 \tag{13}$$

Question 2: What is the dividend yield that is consistent with the answer to the question above?

Using Equations (9) and (12) above and the data in Table 1 above the equation for the DDM adjusted dividend yield is...

$$d = 1+0.12-\left[\left(1+0.05\right)^{5}+0.60\times\left(0.12-0.05\right)\left(1+0.03\right)^{5}\left(1+0.05\right)^{-1}\left(\frac{1.0194-1.0194^{6}}{1-1.0194}\right)\right]^{\frac{1}{5}} = 3.237\% \quad (14)$$

Proof: Using Equations (15), (10) and (14) above total company value at the end of year 5 is...

$$P_5 = 7,500,000 \times \left(1+0.1200-0.03237\right)^{5} = 11,415,000 \quad (15)$$